Finding the operating point of Eulerian flow machines

J. R. Tippetts*

Examples of Eulerian flow machines (EFM) are turbomachines, jet pumps and vortex amplifiers working with incompressible non-cavitating flow. They are 'Eulerian' in the sense used by Paynter[2] in his work on turbomachines subject primarily to dynamic flow forces. Efficient methods are specified in this paper for finding the operating point of an EFM from its characteristics and any two state-defining variables. A trivial example is to find the torque and pressure of a pump when the speed and flow are given. This is simple because the usual constant-speed characterisation favours the solution, but if other pairs of variables are given, the problem is less simple. For jet pumps or the many 'power fluidic' devices the variety of problems is much greater because of combinatorial aspects, although the fluid mechanics is analogous to that of the turbomachine. Solution procedures are specified first for turbomachines; there are six 'algorithms'. For general '3-terminal' EFM (jet-pumps etc) it is shown that there are 108 characterisation formats and that the 30 listed algorithms enable any of them to be solved given any possible variable-pair. Graphical and computational implementations are described

Key words: *turbomachines, power fluidics, fluid mechanics, mathematical modelling*

The type of problem considered here is: given the values of two flows or pressure-drops, find the operating state of a vortex amplifier from its characteristics. A similar but easier task is to find the operating point of a turbomachine given the shaft torque and the pressure-difference.

The vortex amplifier is an example of many power-fluidic devices for which the same problem can occur so general solution methods are needed. The methods should be efficient because, in simulating the operation of a multi-element circuit, the operating point of an element may have to be found many times.

The power-fluidic devices are 3-terminal elements meaning that they have three pipe-connections to the external network and that the flow is incompressible. Examples of these are jet-pumps, venturi-like 'RFD' devices, flow junctions, special Y-joints, turn-up vortex amplifiers, opposed jet amplifiers, and wall-attachment diverters with zero control flows. They are used in fluid-handling systems (not information handling as is often thought in the context of fluidics) as described elsewhere[1].

'Eulerian' has the same meaning as that used by Paynter[2] in describing turbomachines subject to purely dynamic forces and with incompressible flow. The characteristics of such an ideal Eulerian Turbomachine (ETM) are constant when non-dimensionalised in terms of the usual flow and pressure coefficients. Similarly, the ideal Eulerian 3-Terminal Element (E3TE) also has constant non-dimensional characteristics. Collectively these are Eulerian flow machines.

Decoding

The characteristics of a rough pipe are described by, for example, the Colebrook formula. That is, the operation of the pipe is 'encoded' and we could say that finding the 'operating' point of the pipe given, say, the flow, is a matter of 'decoding'; in this case it means finding the pressure-drop. Even for a device as simple as a pipe, decoding is not trivial as can be judged by the effort put into devising easier-to-solve formulae than the Colebrook equation.

For the pipe, only two types of causality need to be considered; given the flow find the pressure-drop and given the pressure-drop find the flow. For the turbomachine or for the 3-terminal element many more forms of causality must be handled by the decoding procedure. Apparently for the turbomachine, decoding is not perceived as a general problem. Probably, this is because the usual constant-speed characteristics suit the majority of cases so that there is no decoding problem; one just reads off the pressure given the flow, for example. When a more awkward problem arises, it is solved one way or another without being regarded as part of a general problem. It is appropriate, therefore, to consider the turbomachine first because it is simpler than the 3-terminal element and because there does not appear to be a general solution procedure.

* Department of Chemical Engineering & Fuel Technology, The University of Sheffield, Mappin Street, Sheffield S1 3JD, UK
Received 18 August 1983 and accepted for publication on 26 October 1983

The Eulerian turbomachine

The ETM is characterised by speed, torque, flow and pressure† (n, t, q and e). Power is often used instead of torque but basically gives the same information and the use of torque gives a closer analogy to the 3-terminal elements.

Characterisation of ETMs

The usual method of characterisation is by plotting e and t as functions of q with n held constant. There is a hierarchy in this allocation of variables. The speed is the primary independent variable, the flow is the second. The pressure and torque are dependent variables with, by general usage, the pressure being the more important. If the turbomachine were not Eulerian, then sets of characteristics would be needed to cover a range of speeds. If it is Eulerian, however, characteristics at one speed can be extrapolated to predict the operation at other speeds. Furthermore, the effect of changes in fluid density and in overall scale of the machine can be found. These effects are usually taken into account by using non-dimensional variables but in the decoding problems considered here, the fluid density and the size of the device are assumed to be known. Therefore, the fully non-dimensional representation is not used. This greatly simplifies the nomenclature and focusses attention on the most important aspect of the decoding problem.

† Pressure means pressure-difference from here on.

Characterising functions

The ETM is considered to be characterised by two functions f_e and f_t relating the four variables according to:

$$\left.\begin{aligned} e &= f_e(q) \\ t &= f_t(q) \end{aligned}\right| \quad n = \text{constant}$$

These could be graphs, as shown in Fig 1, or represented by an array of values in a computer giving p points on the characteristics in the general form

$$\begin{array}{cccc} n_1 & q_1 & e_1 & t_1 \\ n_2 & q_2 & e_2 & t_2 \\ - & - & - & - \\ n_p & q_p & e_p & t_p \end{array}$$

Notation

a, b, c	Either flows or pressure drops in generalised treatment of EFM.
B, C	Non-dimensional variables, ratios of flows or pressure-drops.
e	Pressure-drop through a turbomachine or other device (subscripted).
E	Normalised e, ie e/n^2 for a turbomachine or e/reference pressure-drop for 3-terminal element.
f_e	Characteristic giving e in terms of q with n constant.
f_t	Characteristic giving t in terms of q with n constant.
F_E	Characteristic giving E in terms of Q.
F_T	Characteristic giving T in terms of Q.
F_1, F_2, etc	Functions interrelating various normalised variables.
m	Scaling factor relating similar operating points in terms of a ratio of flows (or speeds for the Eulerian turbomachine).
n	Shaft speed of rotation of a turbomachine.
q	Flow.
Q	Normalised flow, q/n for a turbomachine of q/reference flow for 3-terminal element.
r	Ratio or coefficient formed from all possible pairs of state-defining variables of 3-terminal element.
t	Torque.
T	Normalised t, ie t/n^2.
x, y, z	Either flows or pressure-drops in generalised treatment of 3-terminal element.
X	Normalised value of x, the dependent variable in homogeneous characterisations or the independent variable in hybrid ones.
Y	Normalised value of y, dependent variable in homogeneous characterisations.
Z	Normalised value of z, 'redundant' variable obtained by summation of X and Y.

Subscripts

c	Control port (of a vortex amplifier).
g	Given value of variable, has same meaning when used as second subscript.
n	Newly created value.
o	'Old' or original value.
p	pth row of data in list of p characterised operating points.
x	Control-to-supply pressure-drop.

Abbreviations

EDT	Eulerian data transformation whereby similar operating points are scaled from an original point.
EFM	Eulerian flow machine.
ETM	Eulerian turbomachine.
E3TE	Eulerian 3-terminal element.
va	vortex amplifier.
3TE	3-terminal element such as a jet pump or vortex amplifier.

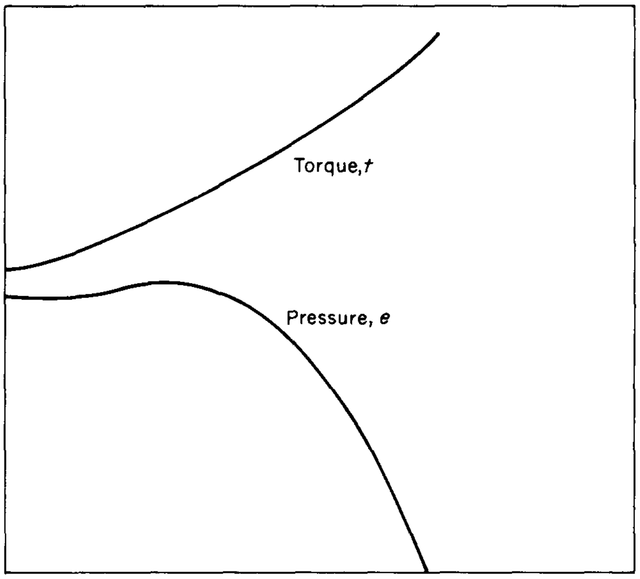

Fig 1 Constant speed pump characteristics

where the entries in the n column could be replaced by a single value in representing the constant-n characteristics.

Eulerian data transformation

A property of the ETM is that if the 'flow-like' variables n and q are both multiplied by a factor m say, and the 'force-like' variables e and t are multiplied by the square of that factor, then the newly created values also represent an operating point of the ETM. If these operating points are denoted 'old' and 'new' (subscripts o and n) then the relationship is given by

$$\begin{array}{lllll} \text{old} & = n_o & q_o & e_o & t_o \\ \text{new} & = mn_o & mq_o & m^2 e_o & m^2 t_o \end{array}$$

The generation of a new operating point, or an array of them, according to this scaling law is an 'Eulerian data transformation' (EDT). For example, the characteristics at an 'old' value of n can be transformed into those at a new value by subjecting each point to an EDT for which m is equal to n_n/n_o.

Analogously this can be represented graphically by plotting the characteristics on double logarithmic graph paper, as shown in Fig 2. If the characteristics are on a movable overlay then shifting the overlay diagonally upwards along a line with a gradient of 2 represents an EDT with an m factor given by the horizontal shift.

Methods of decoding for the ETM

Any two of the four variables n, q, t, e, specify an operating point. The decoding problem is to find the other two variables at that point. The ease with which this is done depends on which variables are given. It is easiest if n and q are given. In general all possible pairs of variables must be considered and there are six such pairs since it is a choice of two-out-of-four.

Method 1

The problem can be solved in various ways and one of these is suggested in an obvious way by the EDT. Suppose n and q are given but n is not the characterised value; the process is as follows:

(1) generate new characteristics by an EDT making n equal to the specified value of n.
(2) Interpolate from the new e and t values those which correspond to the specified value of q.

Step 1 is straightforward, at least for a computer. Step 2 involves searching amongst the values in the q column for those above and below the specified value and then interpolating corresponding values of e and t at this point.

The procedure can be applied for any given pair of variables. Consider as a further example the case when e and t are given:

(1) Generate constant e characteristics with an EDT for which m is defined by;

$$m = \left(\frac{e_n}{e_o}\right)^{1/2}$$

here e_n is the spcified value and e_o represents values in the original column of data. Unlike the case where n was changed, m is recalculated for every data point (ie each row of the array).
(2) Look down the column of t values and interpolate between adjacent points to find q and n.

In this case step 2 is, in effect, an inversion of the scaled function f_t but the significance of this depends on the method of interpolation and on whether f_t is monotonic. If linear interpolation were used then there would be no difference in interpolating for any of the variables but this benefit would have to be weighed against the accuracy.

Critique of Method 1

Method 1 solves the decoding problem rather inefficiently in terms of computational effort. This is because, unless by good fortune n is the same as in the original data, the whole array has to be transformed. For use in simulating a fluid system in which perhaps many hundreds of solutions are needed this inefficiency is a drawback.

An improved method relies on a familiar trade-off in that pre-calculated data requiring more memory can be used to simplify the decoding calculations.

Method 2

Three sets of data are stored in which successively n, q, and e are held constant. Arbitrarily t is left out of this but, mathematically, any of the other variables could have been. These arrays are generated by an EDT operating on the original data. The process produces a total of $9p+3$ values to be stored. (The '3' being the constant values of n, q and e).

The decoding process for a specific variable-pair is:

Values given: e_g and t_g

(1) Form the ratio $m=(e_g/e_c)^{1/2}$ then get the transformed value of t_g:

$$t_n = m^2 t_g$$

(2) Interpolate in the constant-e characteristics to find values of n_n and q_n corresponding to the value of t_n.
(3) Transform n_n and q_n back to give their true values using the scaling factor m:

$$n = n_n/m$$

$$q = q_n/m$$

This then solves the problem without the need for any array transformation.

Other pairs of given variables are treated in a similar way but noting that m is defined directly as a ratio of the variables q and n and as the square root of the ratios of variables e and t.

Only three sets of characteristics have to be stored because a given pair of variables is bound to contain one of the three held constant.

Improvement on Method 2

Further improvement can be explained by reference to the logarithmic plot shown in Fig 2. Consider a specific given variable-pair and the consequent procedure as follows:

given q_g and t_g
(1) Identify the point with coordinates q_g, t_g on the logarithmic grid.
(2) Note the upward diagonal shift needed to make the $q \sim t$ characteristics pass through q_g, t_g. The horizontal shift gives m from which the other values can be found or else they are simply read off from the shifted characteristic.

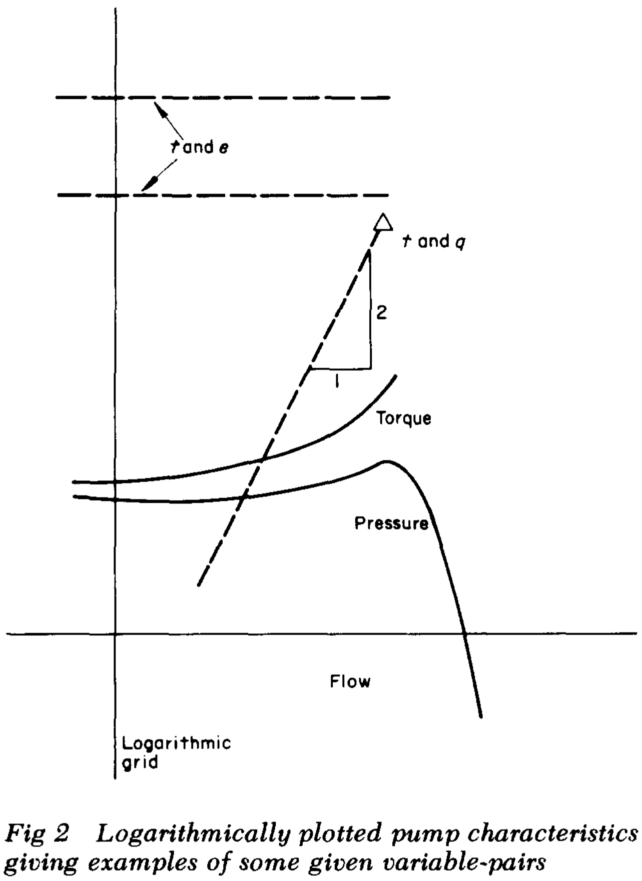

Fig 2 Logarithmically plotted pump characteristics giving examples of some given variable-pairs

The need for a movable overlay is avoided, however, by drawing a line through the point q_g, t_g with a gradient of 2 to intersect the $q \sim t$ characteristic. The horizontal displacement of the intersection point from q_g gives the m factor by which the data at the intersection operating point should be transformed to give the values of e and n.

The procedure is less straightforward for other pairs of variables; for example, consider e and t given:

These values can only be represented by horizontal lines on the logarithmic grid and it would be necessary to shift the overlay until simultaneously these lines intersected the e and t curves at the same value of q, ie with the intersection points vertically aligned. This is not convenient but it should be noted that the vertical distance between e and t represents a ratio of one to the other, and that the aligned intersection points is therefore the coincidence of that ratio in the characterising functions. Suppose the ratio t/e were included as an auxiliary function of q in the plotted characteristics. This would greatly simplify the procedure which is specified as follows:

values given: e_g and t_g
(1) Form t_g/e_g then find this value on the t/e characteristic and the corresponding value of q.
(2) At this value of q find the value of e_o from the e characteristic.
(3) Find the factor m to transform q and n from the characteristics to values at the operating point, ie:

$$m = (e_g/e_o)^{1/2} \quad q = mq_o \quad n = mn_o$$

To implement such a procedure numerically it is necessary to create ratios or coefficients such as t/e, t/q^2 etc from all six variable-pairs which are invariant when subjected to an EDT. Some of these coefficients like t/q^2 (which represents the line on the logarithmic grid through q_g, t_g) are equivalent to already-recognized coefficients, in this case the torque coefficient, but it is useful to make a comprehensive list for the purpose of decoding, as described next.

Method 3

This third and more favoured method for decoding the ETM is described in terms of 'normalised' variables and functions as distinct from the dimensional variables used so far. As a consequence the ETM is considered to be characterised by the functions F_E and F_T as listed below with the coefficients and auxiliary functions:

1. $\dfrac{q}{n} = Q$ normalised independent variable

2. $\dfrac{e}{n^2} = E = F_E(Q)$
3. $\dfrac{t}{n^2} = T = F_T(Q)$

$\Big\}$ normalised characterising functions

4. $\dfrac{e}{q^2} = \dfrac{E}{Q^2} = F_4(Q)$
5. $\dfrac{t}{q^2} = \dfrac{T}{Q^2} = F_5(Q)$
6. $\dfrac{t}{e} = \dfrac{T}{E} = F_6(Q)$

$\Big\}$ auxiliary functions giving the coefficients explicitly in terms of Q

These coefficients include all possible pairs of the variables n, q, e and t and, as required, they are invariant when subjected to EDTs. There is some latitude in their definitions for example $q/e^{1/2}$ could replace coefficient 4 but, arguably, those listed have the simplest formulation. Some coefficients are analogous or equivalent to coefficients used in well-established turbomachine technology, but the objective here is to make a complete list of coefficients in their simplest form for decoding.

The reason for expressing the coefficients 4, 5 and 6 in terms of explicit functions (F_4, F_5 and F_6) of Q is to speed the decoding process by storing extra pre-calculated data. The total array size for p data points is less than that for method 2, ie $6p$ as opposed to $9p+3$.

Specific decoding example for method 3

A specific example will explain the notation used for specifying the general solution:

Given e_g and t_g

(1) Form the ratio t_g/e_g and by inverting F_6 find Q. This process is denoted by F_6^{-1}.
(2) The value of Q is used in the characterising function F_E to give the value of E. This denoted by F_E.
(3) Since E and e are known n is found from the defining equation for E, ie:

$$n=\left(\frac{e_g}{E}\right)^{1/2}$$

This is denoted simply by 'n'.

(4) Since n and Q are known, q is formed from the defining equation for Q, ie:

$$q=nQ$$

This is denoted by 'q'.

The whole procedure can be represented by the sixth algorithm in the following complete list of decoding algorithms:

	Given Variables		Algorithm
1.	n	q	$F_E e \cdot F_T t$
2.	n	e	$F_E^{-1} F_T q t$
3.	n	t	$F_T^{-1} F_E q e$
4.	q	e	$F_4^{-1} F_T n t$
5.	q	t	$F_5^{-1} F_E n e$
6.	e	t	$F_6^{-1} F_E n q$

An obvious feature is that F_4, F_5 and F_6 are required only in their inverted form so, to maximise computational efficiency, they could be stored in this form. Another feature is that F_E occurs more often than F_T but this is because of an arbitrary choice in algorithm 6 where either could be used.

Eulerian 3-terminal elements

The vortex amplifier is an example of a 3-terminal element which highlights the similarities and the difference between 3TE and ETMs.

The variables are defined by

q_s, q_c and q_o	supply and control inflow and outflow from the outlet
e_s	supply-to-outlet pressure drop
e_c	control-to-outlet pressure drop
e_x	control-to-supply pressure drop

Characterisation variables can be chosen by omitting a redundant flow and pressure variable but even for the vortex amplifier the choice varies. In one characterisation format e_s is held constant, e_c is the second independent variable and q_s and q_c are the dependent variables with e_x and q_o omitted (as shown in Fig 3). In another format, e_c is constant, and e_x and q_c are represented as functions of q_s. For elements such as jet pumps, constant flow characteristics are more appropriate and the diversity in characterisation increases as more types of element are considered. Hence, unlike the turbomachine, we cannot assume a standard characterisation format with a fixed allocation of independent-, dependent-, and characterising variables. This, however, affects the required decoding process, so characterisation itself must be considered first.

Enumeration and classification of 3TE characterisations

There are probably infinitely many ways of characterising an element so the following section is concerned only with conventional characterisations

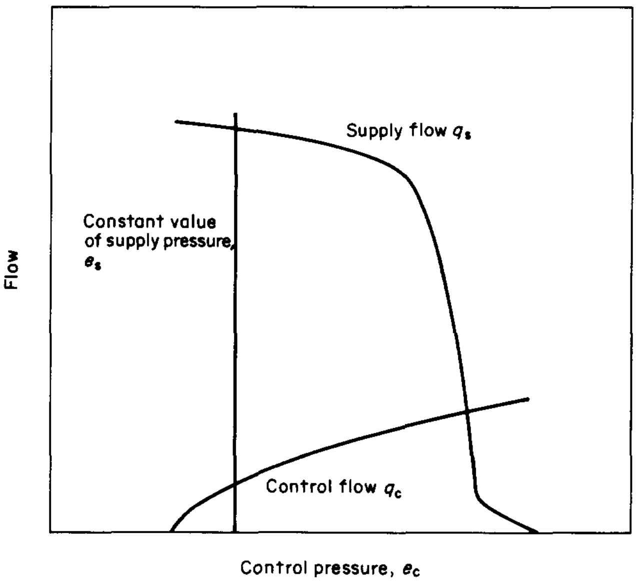

Fig 3 Constant-e_s vortex amplifier characteristics

analogous to those used for ETMs, jet pumps and even transistors. We have to allocate therefore the following types of characterising variables amongst the total number of six flow and pressure differences:

The independent variable held constant
The second, varying independent variable
Two dependent variables

The allocations can be enumerated as follows using the vortex amplifier variables as an example:

Two independent variables can be chosen in 15 ways from the 6 variables. Three of these allocations are pairs of flows and three are pairs of pressures.

Consider the three flow pairs. The independent variables cannot include a flow variable so this constraint limits the allocations of these to six as indicated in the following:

Dependent Variables		Independent Variables Constant	2nd	
q_s q_c	3 choices from the flows	e_s	e_c	6 choices from the pressures
		e_s	e_x	
q_s q_o		e_c	e_s	
		e_c	e_x	
q_c q_o		e_x	e_s	
		e_x	e_c	

The total number of allocations in which both the dependent variables are flows is therefore $3 \times 6 = 18$. This set of allocations can be described as an 'impedance' type of characterisation because the pressure is regarded as a function of the flow. Suppose now that 'pressure' is substituted for 'flow' in the previous set of allocations we get another set of 18 in which both dependent variables are pressures. These are analogous to admittance characterisations.

It is convenient to refer collectively to both sets, 36 allocations in total, as 'homogeneous' since the dependent variables are of one type (as are the independent variables).

The remaining allocations are 'hybrid' with flows and pressures appearing as both dependent and independent variables. The enumeration of these can be done by considering a particular allocation.

Dependent	Constant	2nd
q_s e_s	q_c	e_c

There are (15–6) ways to choose the dependent variables. Having done this, the constant independent variable can be chosen freely from the remaining 4 unselected variables; q_c is chosen in the example. The second independent variable cannot now be a flow because two flows have already been used so there are only the two remaining pressures to choose from, ie e_c or e_x; e_c was actually chosen. The total number of hybrid allocations is therefore given by:

$$(15-6) \times 4 \times 2 = 72$$

So the total number of characterisations is:

$$72 + 36 = 108$$

If a distinction is made between the two dependent variables, ie if one is more important than the other, then this number is doubled. The main point is that any decoding procedure must take account of the diversity in characterisation. The classification of types of characteristics, however, suggests that a properly generalised system of decoding algorithms should be able to deal with any of them.

Some properties of E3TE

Having already discussed the ETM it is convenient to draw analogies between it and the E3TE.

Being characterised by two flows and two pressures, the E3TE is similar to the ETM if the speed and the torque are regarded like a flow and a pressure. Eulerian similitude for the E3TE can be expressed in the same way as for the ETM, ie, if all the flows are multiplied by a factor m and all pressure drops multiplied by m^2 then the newly derived values represent an operating point. Nondimensionalised characteristics are constant and logarithmic plots can be used as described by Tippetts and Royle[3] for solving circuit problems.

Differences arise because a 3-terminal element can be connected into a network in many different ways and its operation can be subject to a wider variety of constraints.

Decoding can be done in various ways similar to the ETM. Method 1 requires that a specified variable is held constant throughout an array, thereby incurring a large number of calculations for each decoded point.

Method 2 for the E3TE requires the storage of 5 arrays of the data each with a different variable held constant. This requires $15p + 5$ memory locations for p data points.

Method 3 requires less storage and is described next.

Decoding method 3 for E3TE

Definition of general variables

To simplify the notation and to allow generality it is convenient to use unsubscripted variables grouped so that pressures and flows can be interchanged according to the two allocations:

1st. Allocation

a b and c are inflows

x y and z are pressure differences

2nd. Allocation

a b and c are pressure differences

x y and z are inflows

A zero-sum sign convention is used so that for both allocations:

$$a + b + c = 0 \quad \text{and} \quad x + y + z = 0$$

No relationship is assumed between the position of the flows and the pressure differences, ie x can be the pressure difference between any two terminals quite irrespective of to which terminal a is an inflow. In all allocations, however:

a is the constant variable.

b is the second independent variable in homogeneous characterisations with x and y being the dependent variables.

x is the second independent variable in hybrid characterisations with b and y being the dependent variables.

c and z are the 'redundant' variables.

Pre-calculated variables and functions

For each variable-pair various coefficients (ratios or simple functions whose values are denoted by $r_1 r_2 - r_n$) are listed in Table 1. Each coefficient (listed in two equivalent forms in the 2nd and 3rd columns) is invariant under an Eulerian data transformation and is given by explicit functions of the 2nd independent variables in the last two columns.

In row 1, B is defined as the normalised independent variable for homogeneous characterisations. It is either a ratio of flows or a ratio of pressure-drops. For hybrid characterisations B is a dependent variable given by the normalised characterising function F_B.

In row 2, C is given as the result of summing pressure or flows. This summation is referred to as a 'Kirchhoffian' relationship which results from the fact that Kirchhoff's laws apply to the 3-terminal element. The calculation is so simple that it is unnecessary to store the result.

In row 3, X is defined as the dependent variable given by the characterising function F_x in terms of B for homogeneous characterisations or as the independent variable for hybrid characterisations. In this case X is either of the form e/q^2 or $q/e^{1/2}$.

Row 4 defines the second characterising functions while row 5 defines the 3rd normalized dependent variable Z. This is obtained by simple summation, ie:

$$Z = -X - Y \quad \text{(this is } F_5\text{)}$$

but Y, at least, is always a dependent variable so that, in decoding, this summation would occur for each data point in the characteristics not just once for each variable-pair. Hence it is justifiable to store the result despite the simplicity of the function F_5.

Rows 6–15 define functions F_6 to F_{15} which give the coefficients explicitly in terms of B or X. The form of the functions are indicated by the normalised representation of r in the third column and are simple function of B, C, X, Y or Z. In fact the functions F_9 and F_{12} are simple summations of other functions ie:

$$F_9 = -F_7 - F_8$$

$$F_{12} = -F_{10} - F_{11}$$

but the results are worth pre-calculating and storing for the same reason as for F_5.

Although there are 15 variable-pairs, not all the functions have to be pre-calculated and stored because Kirchhoffian relationships reduce the number of distinct variable-pairs for decoding. Thus, if the variable pair a, b can be decoded a, c or c, b can also be decoded by first converting these to the variable-pair a, b. For this reason the values of functions F_2, F_6, F_{14} and F_{15} do not have to be precalculated and stored. The total number of storage locations needed is therefore $11p$ for p operating points.

Generalised decoding algorithms for E3TE

Decoding algorithms for each of the 15 possible variable-pairs are listed in Table 2 using the same type of notation as for the ETM. Some extra explanation is appropriate so consider a specific algorithm, say, that in row 11:

Table 1

Variable pair	Coefficient $r_1 r_2 \ldots r_{15}$	Symbol or equivalent form	Functions giving coefficients explicitly: Homogeneous	Hybrid
1 a b	b/a	B	B (ind. var.)	$B = F_B(X)$
2 a c	c/a	C	$C = -B - 1$	
3 a x	$a^n x$	X	$X = F_X(B)$	X ind. variable
4 a y	$a^n y$	Y	$Y = F_Y(B)$	$Y = F_Y(X)$
5 a z	$a^n z$	Z	$Z = F_5(B)$	$= F_5(X)$
6 b c	c/b	C/B	$= F_6(B)$	$= F_6(X)$
7 b x	$b^n x$	$B^n X$	$= F_7(B)$	$= F_7(X)$
8 b y	$b^n y$	$B^n Y$	$= F_8(B)$	$= F_8(X)$
9 b z	$b^n z$	$B^n Z$	etc. to F_{15}: functions giving r-values in terms of B or X.	
10 c x	$c^n x$	$C^n X$		
11 c y	$c^n y$	$C^n Y$		
12 c z	$c^n z$	$C^n Z$		
13 x y	y/x	Y/X		
14 x z	z/x	Z/X		
15 y z	z/y	Z/Y		

Note $n = -2$ if a is flow and $-\frac{1}{2}$ if a is pressure

Table 2 General form of decoding algorithms for E3TE

Given variable pair	Homogeneous	Hybrid
1 a b	$F_x x \cdot F_Y y$	$F_B^{-1} F_Y yx$
2 a c	b, then same as 1	b, then same as 1
3 a x	$F_x^{-1} F_Y yb$	$F_B b \cdot F_Y y$
4 a y	$F_Y^{-1} F_X xb$	$F_Y^{-1} F_B bx$
5 a z	$F_5^{-1} F_X xb$	$F_5^{-1} F_B bx$
6 b c	a, then same as 1	a, then same as 1
7 b x	$F_7^{-1} a F_Y y$	$F_7^{-1} a F_Y y$
8 b y	$F_8^{-1} a F_X x$	$F_8^{-1} F_B ax$
9 b z	$F_9^{-1} a F_X x$	$F_9^{-1} F_B ax$
10 c x	$F_{10}^{-1} Ca F_Y y$	$F_{10}^{-1} a F_Y y$
11 c y	$F_{11}^{-1} Ca F_X x$	$F_{11}^{-1} F_Y ax$
12 c z	$F_{12}^{-1} Ca F_X x$	$F_{12}^{-1} F_B Cax$
13 x y	$F_{13}^{-1} F_X ab$	$F_{13}^{-1} a F_B b$
14 x z	y, then same as 13	y, then same as 13
15 y z	x, then same as 13	x, then same as 13

The given variables are c and y. If they relate to a homogeneous characterisation, and in particular an impedance type (a, b and c being flows) then firstly the coefficient y/c^2 is calculated and the corresponding value of B found by inverting the auxiliary function F_{11}. The value of C is then calculated from the simple Kirchhoffian relationship which then allows a to be found from:

$$a = c/C$$

The characterising function F_x is then used to find X in an explicit function of B. x is then found using:

$$x = Xa$$

At this point two pressures and two flows are known so if the third pressure or flow variables are wanted they can be found easily by a Kirchhoffian relationship. This final step is not listed because it applies to all the algorithms.

Note that other algorithms could have been used. For example, in the second step instead of finding C, the newly found value of B could have been used with F_x to get X and then a from:

$$a = x/X$$

This is a seemingly straightforward sequence but the only way to progress is by evaluating F_Y to give, ultimately, the second pressure variable y. The algorithm would be symbolised by:

$$F_{11}^{-1} F_Y a F_x x \quad \text{(then } y \text{ and } b \text{ found)}$$

But this is less efficient than that listed in the table because evaluating F_Y is more complicated than the summation needed to get C.

For the hybrid characterisation, with a being a pressure for example, the coefficient $y/c^{1/2}$ is formed and then used in F_{11} to find X. This is then used in the characterising function F_Y to give Y which is used to find a from:

$$a = y/Y$$

The value of x is found similarly in the fourth step by:

$$x = aX$$

Finally, Kirchhoffian relationships would give the third pressure and flow variables if needed. This step is not symbolised in the algorithm.

Various comments may be made by considering the whole table of algorithms. Note that in algorithms 2, 6, 14 and 15 Kirchhoffian relationships are used as a first step so that decoding is then done by already-specified algorithms. This does not actually introduce another step because they yield 5 of the 6 variables so only one further step is needed at the end whereas the other algorithms need two. Judicious specification of the sequences means that no more than two stored functions are evaluated in any algorithm and no more than one function is inverted.

Comparison of the homogeneous and hybrid algorithms shows that more homogeneous algorithms have more than 4 steps than the hybrid algorithms. This does not mean that the hybrid characterisation is more efficient because 14 of the hybrid algorithms include the inversion of a function whereas there are only 12 such homogeneous algorithms. Normally this would favour the homogeneous characterisation.

Decoding example

Consider a vortex amplifier described by the admittance type of homogeneous characterisation frequently used. The steps needed in a computer-implemented decoding process are as follows.

Step 1 Characterising data

The significance of the variables and the sign convention for flows and pressure-drops must be given by setting up the following correspondence of variables:

$a = e_s$	constant independent variable
$b = -e_c$	2nd (varying) independent variable
$c = e_x$	3rd redundant pressure variable
$x = q_s$	main dependent variable
$y = q_c$	secondary dependent variable
$z = q_o$	3rd redundant flow variable

Since a, b and c are pressures, the exponent n is equal to $-\frac{1}{2}$.

Raw data is supplied representing p operating points in the form $e_s e_c q_s q_c$ making a total of $4p$ values or $3p+1$ if e_s is accurately constant at one value throughout. In fact, because of the normalised representation of the data created in the pre-calculation stage, e_s does not have to be constant. In principle, therefore, the input data could be randomly scattered points but usually there are reasons for having a fairly orderly form for the data.

Step 2 Pre-calculation

The following coefficients are calculated for each of the p points:

$$B = e_c/e_s$$

$$F_x = q_s/e_s^{1/2}$$

$$F_Y = q_c/e_s^{1/2}$$

$$F_5 = -(q_c + q_s)/e_s^{1/2} = -F_x - F_Y$$

$$F_7 = q_s/e_c^{1/2}$$

$$F_8 = q_c/e_c^{1/2}$$

$$F_9 = -(q_s + q_c)/e_c^{1/2} = -F_7 - F_8$$

$$F_{10} = q_s/(e_c - e_s)^{1/2}$$

$$F_{11} = q_c/(e_c - e_s)^{1/2}$$

$$F_{12} = -(q_c + q_s)/(e_c - e_s)^{1/2} = -F_{10} - F_{11}$$

$$F_{13} = q_c/q_s$$

These would occupy $11p$ memory locations and would enable any variable-pair to be decoded. Obviously, if only specific variable-pairs needed decoding, less of these functions would need to be calculated and stored.

Step 3 Decoding

As an example of the calculations needed to decode a particular variable-pair, consider algorithm 12:

given q_o and e_x

(1) Let $r = q_o/e_x^{1/2}$
(2) Search for r amongst the listed values of F_{12}. Let r_n and r_{n+1} be the values below and above r and B_n, B_{n+1}, X_n and X_{n+1} be the corresponding values of B and X.
(3) Using linear interpolation to find B:

$$\text{let } m = (r - r_n)/(r_{n+1} - r_n)$$

$$B = m(B_{n+1} - B_n) + B_n$$

(4) $c = -B - 1$
(5) $a = c/C$ ie e_s is found
(6) $X = m(X_{n+1} - X_n) + X_n$
(7) $x = aX$ ie q_s is found
(8) $q_c = q_o - q_s$ and $e_c = e_s + e_x$

Using the linear interpolation as indicated in steps 3 and 6, this process needs the following operations:

9 summations
3 multiplications
3 divisions
1 square root
1 search for a value amongst a list of p values.

Other algorithms differ slightly in the number of some of the operations, but this inventory gives a measure of the complexity of the process.

In practice there are other important considerations which are fairly common knowledge. One is the monotonicity of the various functions and how this, or the lack of it, influences the search and interpolation procedures. Suitable ordering of the data to facilitate this could be an important part of the pre-calculations.

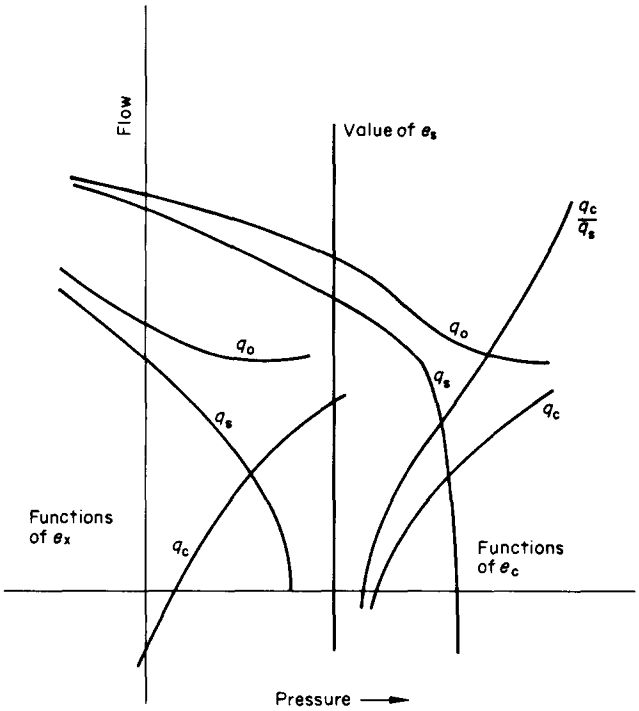

Fig 4 Logarithmically plotted characteristics of vortex amplifier including auxiliary functions

Graphical decoding using logarithmic plots

Decoding can be done graphically as described here for the vortex amplifier.

The characteristics are plotted logarithmically as shown in Fig 4. The basic constant-e_s characteristics are supplemented by five auxiliary plots which altogether give q_s, q_c, q_o and q_c/q_s as functions of e_c and q_s, q_c and q_o as functions of e_x. On a linear plot the functions of e_x would just be horizontally shifted versions of the functions of e_c but on the logarithmic plot they are distorted so they are not redundant. The plotting of these functions is analogous to the pre-calculations done in the computer-implemented scheme.

The whole set of characteristics are considered to be on a movable overlay which can be shifted diagonally along a fixed path at a gradient of $\frac{1}{2}$ on the logarithmic grid. (The scale for the q_c/q_s characteristic must be regarded as fixed to the overlay but all the others have 'disembodied' scales).

Decoding depends slightly on which groups of variables are given. The three following examples cover these different groups.

(1) Suppose the values given are two flows, ie q_{og} and q_{sg}. The third flow q_c is calculated and then the ratio q_c/q_{sg}, which is identified as a point on the unshifted q_c/q_s characteristic. Vertically aligned with this point is a point on the $q_o \sim e_c$ characteristic which is then shifted diagonally with the overlay until its ordinate is equal to q_{og}. The shifted position of the point gives the value of e_c and the value of e_s is given by the new position of the constant-e_s marker (a vertical line on the overlay).
(2) Suppose the values given are e_{xg} and q_{sg}. A point with these coordinates is identified and then the overlay shifted so that the $q_s \sim e_x$ characteristic passes through it. The vertically aligned points on the other 'e_x' characteristics give q_c and q_o. The shifted position of e_s again gives its value.
(3) Suppose the values given are e_{sg} and e_c. The overlay is shifted until e_s has the value e_{sg} and then the values of q_s, q_c and q_o are read-off from the 'e_c' characteristics at the value e_{cg}.

If the characteristics are not on a movable overlay, the effect of shifting can be synthesised by drawing sequences of lines either vertically, horizontally or at a gradient of $\frac{1}{2}$.

Similar graphical methods can be devised to cope with other devices and characterisations.

Non-Eulerianarity

In practice the range within which Eulerian Similitude prevails is bounded by functions of Reynolds number and Mach or Cavitation number. Consequently these constraints would generally have to be included in the overall characterisation and decoding scheme but this is a familiar aspect of fluid mechanics so it has not been described.

Conclusion

When many operating points of an ETM must be found, the decoding algorithms given earlier are recommended. Most, if not all, of these are represented by various methods scattered in the existing literature of turbomachines but here they are gathered together and put in a computationally efficient form. The coefficients in the section describing method 3 are identical or equivalent to quantities which also occur in the literature, but for the purpose of decoding they have been put in the simplest form. This shows that their origin is strongly based on combinatorial features of the relationships governing the ETM in addition to the well known relationships which ultimately derive from Newton's laws.

For the 3-terminal elements, although the fluid mechanics is no more complicated than for the ETM, the combinatorial relationships *are* more complicated.

Since there is no universal way of characterisation, the total number of characterisations was investigated and shown to be 108, or 216 if the dependent variables are treated as distinct. Despite this diversity, only two classes of characterisation need to be distinguished and so, by using coefficients analogous to those for the ETM, a comprehensive set of decoding algorithms could be specified as given in Table 2.

References

1. **Tippetts J. R., Priestman G. H. and Thompson D.** Developments in Power Fluidics for Application in Nuclear Plant. *ASME J. Dyn. Systems Measurement and Control, Dec. 1981,* **103**, *342–351*
2. **Paynter H. M.** The Dynamics and Control of Eulerian Turbomachines. *ASME J. Dyn. Systems Measurement and Control, Sept. 1972,* **94**, *198–205*
3. **Tippetts J. R. and Royle J. K.** Design of Flow Control Circuits Involving Unvented Bistable Amplifiers. *Fluidics Quarterly, Oct. 1971, 3(4), 1–15*

BOOK REVIEW

Numerical Properties and Methodologies in Heat Transfer

Ed. T. M. Shih

This handsome volume is the first Proceedings publication in the series in 'Computational Methods in Mechanics and Thermal Sciences', with W. J. Minkowycz and E. M. Sparrow as series editors. It is the final 'reviewed and revised' form of the papers for the US Second National Symposium on Numerical Methods in Heat Transfer, held in Mayland in September 1981. The book comprises thirty-two papers including six invited reviews.

In numerical heat transfer one may now discern two parallel trends: the development of alternative methods and application to increasingly complex problems. This publication is particularly rich in the first aspect, this being evident not just in the fine review papers and specific method studies. The final paper on one-dimensional enclosed flames, for example, gives a thorough comparison of no less than nine, mainly finite-difference, methods for that problem.

The review papers are authoritative and satisfying. They are also readable for those with some prior knowledge of the subject. The introductory trio survey respectively finite-difference methods for parabolic equations, variational principles, and a finite-difference/finite element comparison. They form a well-chosen and well-balanced overview of alternative methods. The other reviews cover singular perturbation problems, multi-phase flow phenomena and radiation.

Applications include free convection in enclosures, external flows, two-phase flows, radiation, and fires and combustion. Here, perhaps, the book is not so strong since the number of papers is rather limited in a given topic. In this and other respects the philosophy and impact of the book are complementary to those of the proceedings of the biennial numerical methods conferences originating from University College, Swansea, U.K.

The title, then, fairly reflects the emphasis of the book. The cost, $69.50, is not unreasonable for a quality production containing much material. It forms a source book for knowledge of available methods for different heat transfer situations. Here, perhaps, an (obvious) proviso should be made. While the study of method is essential, it is of necessity done for the simpler problem, and the desired application introduces variables which themselves can affect the numerical properties of the method. However, the very number of alternatives possibly contains the necessary warning: in leaving the familiar solved problem for the complex unknown, the choice of method must be made with care. D. Brian Spalding's comment in a specific context surely has some generality: 'Proceed cautiously; you are on your own'. I feel this book is a good reflection both of the current potential of numerical methods in heat transfer, and of the effort needed in their application.

M. W. Collins
The City University, London, UK

Published by Hemisphere/Springer-Verlag. Hemisphere Publishing Corporation, Berkeley Building, 19 W 44th Street, New York NY 10036, USA, or Springer-Verlag, Heidelberger Platz 3, Postfach, D-1000 Berlin 33, FRG